

# Effects of Wind on Aircraft Cruise Performance

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Analytic expressions are developed that predict the improvements in range, flight time, and fuel consumption that can be achieved by appropriate corrections to the no-wind best-range airspeed. Input data required are type of power plant, no-wind best-range airspeed, and magnitude of the headwind or tailwind component. Application of these expressions to a series of typical aircraft, ranging from a wide-body turbofan to a single-engine piston-prop, shows that the possible fuel savings are such that the effects of wind on fuel consumption might warrant more consideration in flight planning.

## Nomenclature

$BPR$	= turbofan bypass ratio
$C_L$	= lift coefficient
$c$	= thrust specific fuel consumption
$\bar{c}$	= horsepower specific fuel consumption
$D$	= aerodynamic drag force
$E$	= lift-to-drag ratio = $L/D$
$K$	= lift-dependent drag coefficient factor
$k$	= conversion factor from horsepower to thrust power
$L$	= aerodynamic lift force
$m$	= relative airspeed parameter = ratio of the actual true airspeed to the no-wind best-range true airspeed
$P$	= thrust power = $TV$
$R$	= relative range = ratio of the actual range to the range for the no-wind best-range airspeed
$V$	= true airspeed
$V_g$	= ground speed
$V_w$	= headwind or tailwind component of the wind velocity
$W$	= aircraft weight
$WF$	= wind fraction = $V_w/V_{br}$
$X$	= cruising range
$\Delta W_f$	= fuel used during cruise
$\eta_p$	= propeller efficiency
$\rho$	= atmospheric density
$\zeta$	= cruise-fuel weight fraction = $\Delta W_f/W$

## Subscripts

$br$	= no-wind best-range conditions
$c$	= corrected for wind
$md$	= minimum-drag
$u$	= uncorrected for wind
$w$	= wind conditions

## Introduction

THE current interest in fuel conservation warrants an examination of the effects of wind upon the cruising performance of fixed-wing aircraft with emphasis on the fuel consumption. Earlier treatments<sup>1,2</sup> of wind effects have used graphical techniques and have not been complete. This paper develops analytic expressions that are reasonably simple and are based on the no-wind best-range airspeed of a particular aircraft or class of aircraft. A parabolic drag polar is assumed, crosswind components are neglected, and only cruise-climb flight is considered. Expressions for the best-range airspeed and relative range in the presence of a constant headwind or tailwind component are derived individually for turbojet and for piston-prop aircraft. These expressions are then used to obtain common relationships for the relative flight time, the relative fuel consumption, and the actual fuel savings. The results obtained for turbojets and piston-props are then extended to turboprops and turbofans.

## Turbojet Best-Range Conditions

The governing equations for quasi-steady flight with a small or zero climb angle are

$$T=D; L=W; \frac{dX}{dt} = V_g \quad (1)$$

where  $V_g$  is the ground speed. Since the fuel flow rate of a turbojet engine is proportional to the thrust, the weight balance equation of the aircraft can be written as

$$\frac{-dW}{dt} = cT \quad (2)$$

where  $c$ , the thrust specific fuel consumption, is assumed to be a function of the altitude only. If an expression for the instantaneous (or point) range is obtained from Eqs. (1) and (2) and then integrated from start to end of cruise with  $C_L$  and  $V$  held constant, the Breguet range equation is

$$X = \frac{V_g E}{c} \ln \left( \frac{1}{1-\zeta} \right) \quad (3)$$

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where  $E$  is the flight lift-to-drag ratio and  $\zeta$ , the cruise-fuel weight fraction, is the ratio of the weight of cruise fuel to the initial gross weight of the aircraft.

Considering only headwind and tailwind components, the ground speed can be expressed as

$$V_g = V \left( 1 \pm \frac{V_w}{V} \right) \quad (4)$$

where  $V$  is the true airspeed, and the plus and minus signs denote a tailwind and headwind, respectively. At this point, a relative airspeed parameter,  $m$ , is defined as

$$m = \frac{V}{V_{br}} \quad (5)$$

where  $V_{br}$ , the no-wind best-range airspeed with a parabolic drag polar, is given by<sup>3,4</sup>

$$V_{br} = \left[ \frac{2(W/S)}{\rho} \right]^{1/2} \left[ \frac{3K}{C_{DO}} \right]^{1/4} = [3]^{1/4} V_{md} \quad (6)$$

where  $V_{md}$  is the minimum-drag airspeed of the aircraft.

Using the airspeed parameter, the lift-to-drag ratio can be expressed as

$$E = 2\sqrt{3} \left[ \frac{m^2}{3m^4 + 1} \right] E_{\max} \quad (7)$$

and the ground speed as

$$V_g = V_{br} \left[ m \pm \frac{V_w}{V_{br}} \right] \quad (8)$$

Consequently, a generalized version of the Breguet range equation that acknowledges the presence of a wind can be written as

$$X = \frac{2\sqrt{3} V_{br} E_{\max} [m \pm (V_w/V_{br})]}{c} \times \left[ \frac{m^2}{3m^4 + 1} \right] \ln \left( \frac{1}{1 - \zeta} \right) \quad (9)$$

Maximizing the range with respect to the true airspeed, by setting the first derivative of Eq. (9) with respect to  $m$  equal to zero, yields the condition that

$$m_{br} = \left[ \frac{m_{br} \pm (2/3)(V_w/V_{br})}{m_{br} \pm 2(V_w/V_{br})} \right]^{1/4} \quad (10)$$

where the subscript  $br$  indicates best-range conditions. When the wind component  $V_w$  is set equal to zero,  $m_{br}$  becomes unity and the no-wind best-range airspeed is indeed equal to the  $V_{br}$  defined by Eq. (6). When  $V_w$  is not equal to zero,  $m_{br}$  is the ratio of the best-range airspeed in the presence of a wind to the no-wind best-range airspeed. This ratio, or relative best-range airspeed, is shown in Fig. 1 for various values of the wind fraction ( $V_w/V_{br}$ ). It can be seen that the increase in airspeed required for a headwind is not only disproportionately larger than the decrease for a tailwind but also increases at a rapid rate, whereas the airspeed for a tailwind levels off, approaching as a limit the value of 0.76  $V_{br}$ , which is the minimum-drag airspeed.

If  $m$  is set equal to unity in Eq. (9), it is possible to define  $X_{brwu}$  as the uncorrected best range in the presence of a wind, where

$$X_{brwu} = \frac{\sqrt{3} V_{br} E_{\max}}{2c} \left[ 1 \pm \left( \frac{V_w}{V_{br}} \right) \right] \ln \left( \frac{1}{1 - \zeta} \right) \quad (11)$$

When  $m$  is other than unity, Eq. (9) can be thought of as the range corrected for wind and given the symbol  $X_{wc}$ . Obviously, when  $m$  takes on the appropriate value of  $m_{br}$  for a given wind fraction ( $V_w/V_{br}$ ),  $X_{wc}$  becomes  $X_{brwc}$ . Defining the relative range,  $R$ , as the ratio of  $X_{wc}$  to  $X_{brwu}$  for a given fuel weight,  $R$  can be expressed as

$$R = \frac{X_{wc}}{X_{brwu}} = 4 \left[ \frac{m^2}{3m^4 + 1} \right] \left[ \frac{m \pm (V_w/V_{br})}{1 \pm (V_w/V_{br})} \right] \quad (12)$$

When  $m = m_{br}$ ,  $R$  represents the best relative range or the maximum improvement in range that can be obtained by correcting the no-wind best-range airspeed for the presence of a wind. Figure 1 shows that the improvement in range with a tailwind is slight, being less than 2% for a tailwind fraction of +0.5, and calls for a 10% reduction in airspeed with an accompanying increase in the flight time. The improvement in range with a headwind is appreciably larger but does not become significant until the wind fraction becomes of the order of -0.3 or larger. For example, if  $V_w/V_{br} = -0.4$ , a 6.4% increase in range can be attained by increasing the no-wind best-range airspeed by 22%.

It may not be possible or desirable to fly at the appropriate value of  $m_{br}$ . Figure 2 is a plot of the relative range  $R$  as a function of the relative airspeed for various values of the wind fraction, with emphasis on headwinds. Examination of the headwind curves shows that: 1) the relative range is relatively insensitive to  $m$  in the vicinity of  $m_{br}$ ; 2) for larger values of the headwind fraction, significant improvements in range can be attained for values of  $m$  less than  $m_{br}$ ; and 3) flying at values of  $m$  less than unity introduces range penalties that increase rapidly as the wind fraction increases. Looking at the single tailwind curve, we see that decreasing the airspeed too much, say to maintain a block time, results in a rapidly increasing range penalty.

It must not be forgotten that the baseline for the relative range is the uncorrected best range in the presence of a wind and not the no-wind best range. In order to compare the effect of the wind on the no-wind best range, multiply the relative range by  $[1 + V_w/V_{br}]$ . Thus, a relative range of 1.25 for a headwind fraction of -0.6 indicates a corrected best range of 0.5 of the no-wind best range. If there were no correction for the headwind, the relative range would be unity and the range would be 0.4 of the no-wind best range.

Before examining the effects of an airspeed correction upon the flight time and fuel consumption for a given range, the best-range conditions for piston-prop aircraft will be established.

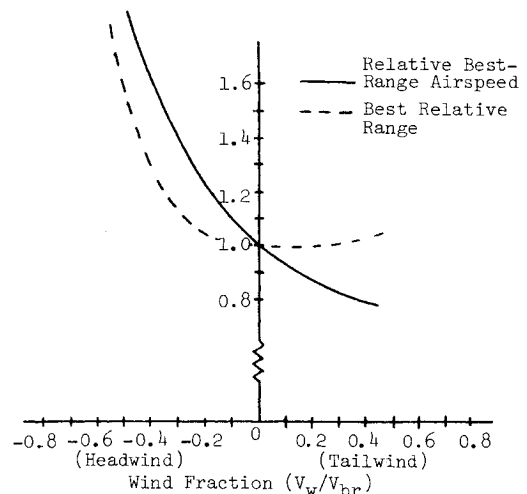


Fig. 1 Relative best-range airspeed and best relative range for a turbojet.

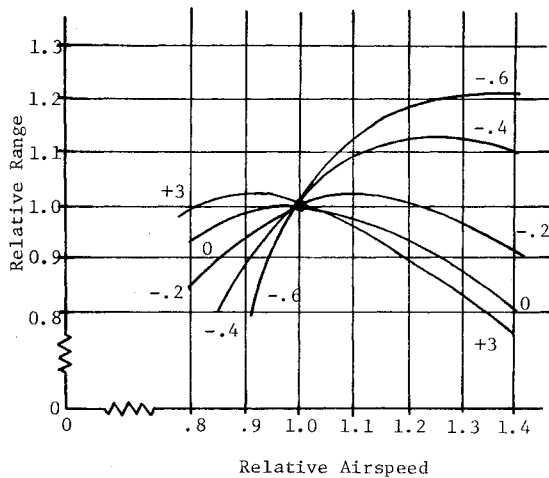


Fig. 2 Relative range as a function of the relative airspeed and wind fraction for a turbojet.

### Piston-Prop Best-Range Conditions

The relationships of Eq. (1) are also valid for a piston-prop aircraft, but the fuel flow rate is proportional to the brake horsepower. Therefore, the weight balance equation for a piston-prop can be written as

$$\frac{-dW}{dt} = \frac{\bar{c}P}{\eta_p k} \quad (13)$$

where  $P$ , the thrust power, is the product of the thrust and true airspeed ( $TV$ ),  $\eta_p$  is the propeller efficiency,  $k$  is a horsepower conversion factor, and  $\bar{c}$  is the horsepower specific fuel consumption. Combining Eqs. (1, 4, and 13) and integrating, yields the Breguet range equation in the form

$$X = \frac{\eta_p k E [1 \pm (V_w/V)]}{\bar{c}} \ln \left( \frac{1}{1 - \zeta} \right) \quad (14)$$

Since the no-wind best-range airspeed for a piston-prop is the minimum-drag airspeed,<sup>3</sup> the relative airspeed parameter is defined as

$$m = \frac{V}{V_{md}} \quad (15)$$

where

$$V_{md} = \left[ \frac{2(W/S)}{\rho} \right]^{1/2} \left[ \frac{K}{C_{DO}} \right]^{1/4} \quad (16)$$

Consequently, the lift-to-drag ratio can be expressed as

$$E = \left[ \frac{2m^2}{m^4 + 1} \right] E_{\max} \quad (17)$$

and the range equation in the presence of a wind becomes

$$X = \frac{2\eta_p k E_{\max} [m \pm (V_w/V_{md})]}{\bar{c}} \left[ \frac{m}{m^4 + 1} \right] \ln \left( \frac{1}{1 - \zeta} \right) \quad (18)$$

Maximizing the range with respect to  $m$  results in the best-range condition that

$$m_{br} = \left[ \frac{2m_{br} \pm (V_w/V_{md})}{2m_{br} \pm 3(V_w/V_{md})} \right]^{1/4} \quad (19)$$

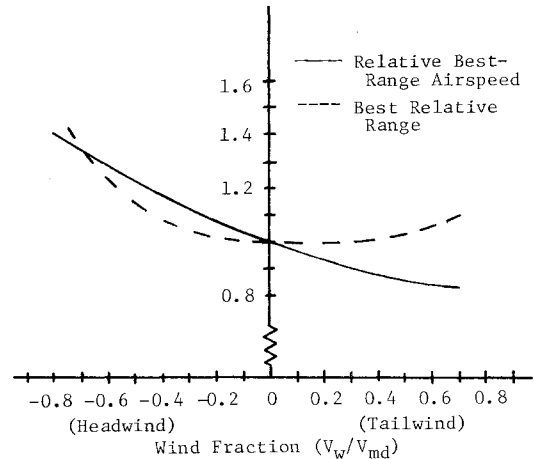


Fig. 3 Relative best-range airspeed and best relative range for a piston-prop.

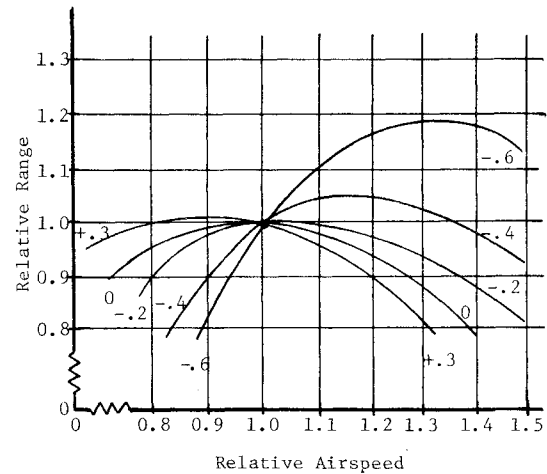


Fig. 4 Relative range as a function of the relative airspeed and wind fraction for a piston-prop.

Setting  $V_w$  equal to zero produces a no-wind  $m_{br}$  of unity, corroborating the selection of  $V_{md}$  as the no-wind best-range airspeed for a piston-prop.

Again, using the uncorrected best range as the baseline, the relative range is

$$R = 2 \left[ \frac{m}{m^4 + 1} \right] \left[ \frac{m \pm (V_w/V_{md})}{1 \pm (V_w/V_{md})} \right] \quad (20)$$

Figure 3 shows both the relative best-range airspeed and the best relative range as a function of the wind fraction, and Fig. 4 shows the relative range as a function of the relative airspeed for several values of the wind fraction. Comparison with Figs. 1 and 2 shows similar relationships with corresponding values for the piston-prop generally lower than those for the turbojet. The limit for the airspeed with an increasing tailwind is  $0.76 V_{md}$ , i.e., the minimum-power airspeed.

### Flight Time and Fuel Consumption

The results of the preceding sections will now be applied to a fixed range, such as an airline route segment, in order to determine the effects of wind on both the flight time (and thus the block time) and the fuel consumption. The relative flight time ( $RFT$ ) is defined as the ratio of the flight time with the airspeed corrected for the wind, to the flight time with the no-wind best-range airspeed. An expression for the  $RFT$  that is

applicable to both a turbojet and a piston-prop is

$$RFT = \left[ \frac{1 \pm WF}{m \pm WF} \right] \quad (21)$$

where  $WF$  is the generalized wind fraction, being equal to  $V_w/V_{br}$  for a turbojet and to  $V_w/V_{md}$  for a piston-prop. The relative airspeed parameter in Eq. (21) is that appropriate for the type of aircraft; i.e.,  $m$  is equal to  $(V/V_{br})$  for a turbojet and to  $(V/V_{md})$  for a piston-prop. Figure 5 shows the  $RFT$  as a function of the relative airspeed for various wind fractions, again with emphasis on headwinds.

The relative fuel consumption ( $RFC$ ) is defined as the ratio of the cruise-fuel weight fraction with a corrected airspeed ( $\xi_c$ ) to the cruise-fuel weight fraction with the no-wind best-range airspeed ( $\xi_u$ ). The expression for the  $RFC$ , again applicable to both turbojets and piston-props, is

$$RFC = \frac{\xi_c}{\xi_u} = \left[ \frac{1 - (1 - \xi_u)^{1/R}}{\xi_u} \right] \quad (22)$$

In Eq. (22), the value of  $R$  to be used in the exponent is obtained from Eq. (12) or Fig. 2 for a turbojet, and from Eq. (20) or Fig. 4 for a piston-prop. Figure 6 shows the  $RFC$  as a function of the relative range for two values of the uncorrected cruise-fuel weight fraction; the larger the weight fraction, the greater the actual range is.

The fuel fraction savings ( $\Delta\xi$ ) and the actual fuel savings ( $W_{fs}$ ) are of greater interest than the  $RFC$  and can be found from the relationships

$$\Delta\xi = \frac{W_{fs}}{W_l} = \xi_u (1 - RFC) \quad (23)$$

where  $W_l$  is the gross weight of the aircraft at the start of cruise. A negative  $\Delta\xi$  represents an increase in the fuel consumption rather than a savings. Figure 7 shows that the greater the range (as indicated by the larger values of  $\Delta\xi$ ), the larger the fuel fraction savings. Equation (23) also shows that the larger the initial cruise weight, the greater are the actual fuel savings.

### Maximum Endurance Conditions

Referring to Eqs. (1) and (2), the instantaneous endurance for a turbojet can be written as

$$\frac{dt}{-dW} = \frac{1}{cT} = \frac{E}{cW} \quad (24)$$

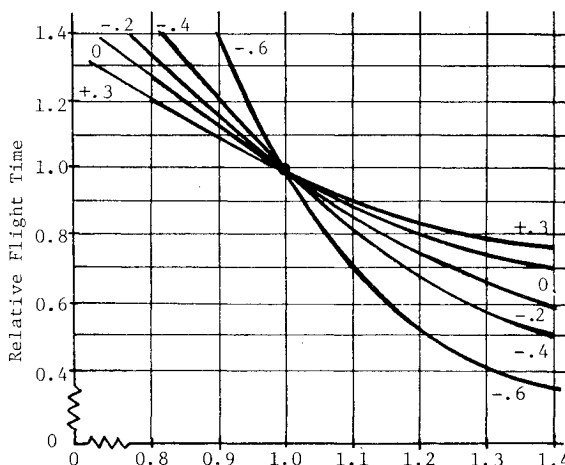


Fig. 5 Relative flight time as a function of the relative airspeed and wind fraction.

which when integrated for constant lift coefficient flight yields

$$t = \frac{E}{c} \ln \left( \frac{1}{1 - \xi} \right) \quad (25)$$

Since neither the groundspeed nor the wind velocity appears, explicitly or implicitly, in either of these equations, the presence of a wind does not affect the endurance. It is also apparent from Eq. (25) that the maximum-endurance airspeed is the minimum-drag airspeed.

The instantaneous endurance of a piston-prop can be expressed, using Eqs. (1) and (13), as

$$\frac{dt}{-dW} = \frac{\eta_p k}{\bar{c} P} = \frac{\eta_p k E}{\bar{c} V W} \quad (26)$$

where  $V$  is the true airspeed. Integration for cruise-climb flight results in

$$t = \frac{\eta_p k E}{\bar{c} V} \ln \left( \frac{1}{1 - \xi} \right) \quad (27)$$

As with the turbojet, the endurance of a piston-prop is unaffected by the presence of a wind. It can be easily shown that the maximum endurance airspeed is the minimum-power airspeed, which is equal to  $V_{md}/(3)^{1/4}$ .

### Turboprops and Turbofans

Turboprops and turbofans combine to varying degrees the characteristics of both the turbojet and the piston-prop. Whereas the turboprop is primarily a propeller aircraft with

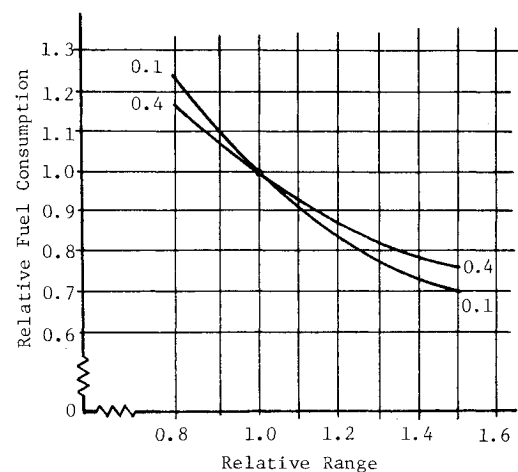


Fig. 6 Relative fuel consumption as a function of the relative range and the uncorrected cruise-fuel weight fraction.

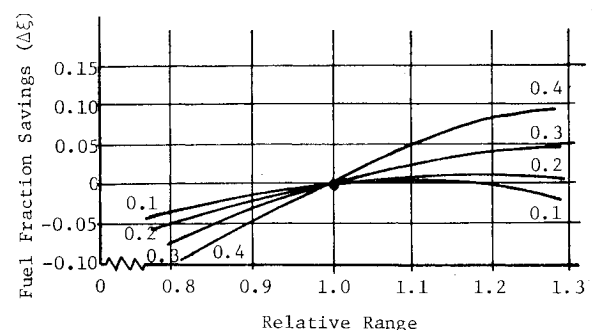


Fig. 7 Relative fuel savings as a function of the relative range and the uncorrected cruise-fuel weight fraction.

the jet thrust of the order of 15% or less of the propeller thrust, the turbofan is more difficult to characterize. When the bypass ratio (*BPR*) is zero, the turbofan is a pure turbojet but as the *BPR* is increased, the turbofan more and more resembles the turboprop. For example, a theoretical turbofan with a *BPR* of 10.4 could have a jet thrust of the order of 17% of the fan thrust.

In view of the similarity of the relative airspeed and range curves for turbojet and piston-prop aircraft, no attempt has been made to develop specific relationships for turboprops and turbofans. Instead, the turbojet and piston-prop solutions will serve as upper and lower limits, respectively, using wind fractions based on the manufacturer's posted no-wind best-range airspeed. For turboprops, the piston-prop values can be used with little loss in accuracy but turbofans will require some judgment, based on the *BPR*, as to the values to be used.

### Results and Examples

In the presence of a tailwind, the difference between the corrected performance of turbojet and propeller aircraft is not sufficient to warrant distinguishing between them. Furthermore, the improvement in range is small, being of the order of 2% for a wind fraction of 0.5 and is accompanied by an airspeed reduction of approximately 10%. For a fixed range such an improvement increases the flight time by 6 to 7% and, for an uncorrected  $\zeta$  of 0.2, produces a relative fuel consumption of approximately 0.98 and a  $\Delta\zeta$  of the order of  $3.5 \times 10^{-3}$ . If the gross weight of the aircraft at start of cruise is 667,300 N (150,000 lb), the resultant fuel savings (or payload increase) is 2335 N (525 lb). As  $\zeta_u$  (the range) and the gross weight increase, the fuel savings also increases. With a  $\zeta_u$  of 0.4, the *RFC* increases slightly but  $\Delta\zeta$  increases, becoming of the order of  $6 \times 10^{-3}$ , which with an initial gross weight of  $2.67 \times 10^6$  N (600,000 lb) produces a fuel savings of 16,015 N (3600 lb).

With headwinds, the improvement in range (or reduction in  $\zeta$  for a given range) does not become significant until the wind fraction begins to exceed 0.3, and requires relatively large increases in the airspeed. Long-range aircraft are characterized by large wind loadings and high cruising altitudes, resulting in theoretical no-wind best-range airspeeds that approach and even exceed the drag-rise Mach number. In the latter case, one design technique is to specify the cruise airspeed, say at  $M=0.8$ , and then select an altitude that maximizes the range by maximizing the lift-to-drag ratio so that the no-wind best-range airspeed approaches the minimum-drag airspeed. With such high no-wind best-range airspeeds, not only is it impossible to increase the airspeed to the best relative airspeed but also the wind fractions will tend to be low, even for the strongest winds.

Consider a current wide-body turbofan with a no-wind best-range Mach number of 0.8 (851 km/h or 528 mph) and a drag-rise Mach number of 0.85; the relative airspeed is 1.06. Let the headwind component be 170 km/h (106 mph) for a wind fraction of  $-0.2$ , which calls for a best-range relative airspeed for a turbojet (from Fig. 1) of 1.085 and for a propeller aircraft (from Fig. 3) of 1.06. Using the  $m$  of 1.06 and making a crude allowance for a *BPR* of 6, Figs. 2 and 4 indicate a relative range of the order of 1.008. For a fixed range, the flight time will be reduced by approximately 7%, and for a long-range mission with  $\zeta_u$  equal to 0.35, the reduction in  $\zeta$  will be  $2.23 \times 10^{-3}$ , which with an initial gross weight of  $2.67 \times 10^6$  N (600,000 lb) produces a fuel savings of 5954 N (1338 lb), or 0.6% of the uncorrected fuel consumption.

General aviation aircraft are characterized by lower wing loadings and lower no-wind best-range airspeeds, shorter ranges, and much lower gross weights than commercial aircraft. For example, a current corporate turbojet with a gross weight of the order of 88,964 N (20,000 lb) has a no-

wind best-range airspeed of approximately 746 km/h (463 mph) and a maximum-cruise airspeed of 852 km/h (528 mph). If the headwind component is still 170 km/h (106 mph), the wind-fraction is  $-0.23$  and the uncorrected best-range groundspeed is 576 km/h (357 mph). Using the appropriate value of  $m_{br}$ , which is 1.10, the corrected best-range airspeed and groundspeed are 820 km/h (509 mph) and 650 km/h (403 mph), respectively, and the best relative range is 1.014. For a fixed range, the flight time is reduced by 11%; and if the uncorrected cruise-fuel fraction is 0.2, the actual fuel savings will be of the order of 219 N (49 lb) or 1.2% of the uncorrected fuel consumption.

Now consider a twin-engine turboprop with a gross weight of the order of 53,378 N (12,000 lb) and a no-wind best-range airspeed of 403 km/h (250 mph). For the same headwind component of 170 km/h (106 mph), the wind fraction is  $-0.42$  and  $m_{br}$  is approximately 1.16. Consequently, the corrected airspeed and groundspeed are 467 km/h (290 mph) and 297 km/h (184 mph), and the relative range is approximately 1.053. The flight time for a fixed range will be reduced by 21%, and with a typical uncorrected cruise-fuel fraction of 0.2, the fuel savings will be of the order of 482 N (108 lb) or 4.5% of the uncorrected fuel consumption.

The final example is a single-engine piston-prop with a gross weight of 15,120 N (3400 lb), a no-wind best-range airspeed of 251 km/h (156 mph), and a maximum cruising airspeed of 322 km/h (200 mph). With a headwind component of 170 km/h, the wind fraction is  $-0.68$  so that  $m_{br}$  is of the order of 1.33, and the corrected airspeed should be 334 km/h. Limiting the airspeed to the maximum cruising airspeed of 322 km/h reduces  $m$  to 1.28 and the corresponding relative range is 1.30. The flight time will be reduced by 47% and with an uncorrected cruise-fuel fraction of 0.1, the fuel savings will be approximately 337 N (75 lb) or 22%.

### Discussion of Assumptions

Numerical methods were used to assess the effects of neglecting the wind angle upon both the relative airspeed parameter,  $m$ , and the relative range. The instantaneous range was expressed as a function of an unspecified  $m$ , the wind and crab angles, and a non-directional wind fraction using the magnitude of the wind vector. A search technique was employed to find the values of  $m$  that maximized the instantaneous range for varying angles and for varying wind fractions. The best relative range was then evaluated over the range of possible wind angles and wind fractions and compared with those obtained considering only the head and tailwind components. As might be expected, the maximum errors were found to occur in the region of direct crosswinds but were less than 1.5% for a wind fraction of 0.5. A sensitivity analysis was also performed and confirmed the results of the simplified analysis.

The implicit assumption of a constant headwind or tailwind component ignores the variations of the wind velocity with range and altitude. The variations with range can be handled by updating the wind fraction and then determining the corresponding effects on the desired airspeed and performance parameters. Inclusion of the altitude dependence of the wind, however, leads to trajectory optimization,<sup>5,6</sup> which is neither relevant to nor within the scope of this paper. Furthermore, such optimizations have significance only for short-range flights, in which climb and descent phase considerations are important.

Although the Breguet range equation as given in Eq. (3) is not exact, either for cruise-climb flight or for constant-altitude flight, the use of more precise formulations does not significantly or qualitatively affect the results. In the case of cruise-climb flight the exact no-wind best-range airspeed will be slightly higher<sup>7</sup> so that the wind fraction will be correspondingly lower. For constant-altitude flight with the no-wind airspeed continually being decreased so as to

maintain a constant lift coefficient, the corresponding wind fraction will be continuously increasing. For constant-altitude and constant-air-speed flight, there are three cases to be considered: no air-speed changes, periodic airspeed changes, and stepped-altitude flight. With no airspeed changes, it can be shown that the best-range relative airspeed becomes a function of the cruise-fuel weight fraction. The periodic airspeed change case approaches in the limit the constant-altitude constant-lift coefficient case previously mentioned and the stepped-altitude flight program approaches cruise-climb flight. In all cases, appropriate corrections to the no-wind best-range airspeed yield similar results to those obtained by the use of Eq. (3).

In real engines the specific fuel consumption is a function of the Mach number and thrust as well as of the altitude. The assumption of the altitude dependence alone implies not only that the aircraft is operating at a specific engine design point but also that the specific fuel consumption curve is essentially flat at that point. To account properly for the very important changes in the specific fuel consumption for off-design point operation requires numerical solutions.

The last assumption to be discussed is that of the parabolic drag polar, which can be considered valid only up to the drag-rise Mach number. Examination of the possible advantages of increasing the airspeed beyond the drag-rise Mach number must be done numerically.

### Concluding Remarks

Since the endurance of an aircraft is unaffected by the wind, there is no need to correct the no-wind airspeed. The range, however, is obviously affected by the wind, beneficially by a tailwind and adversely by a headwind. The range can be improved by decreasing the airspeed with a tailwind and increasing it with a headwind; an improvement in range translates into a fuel savings and payload increase for a fixed range.

With a knowledge of the magnitude of the tailwind or headwind component and of the no-wind best-range airspeed of an aircraft, it is possible to determine the effects of a wind upon the range performance and the airspeed corrections to be made. For a given wind fraction, a turbojet requires larger airspeed changes and has larger improvements in range than

do the piston-prop and turboprop. The turbofan falls somewhere in between, depending upon the value of the *BPR*.

In the presence of a tailwind, the improvement in range (fuel savings) is small, being less than 2% for a wind fraction of +0.5 and is accompanied by a 10% reduction in the airspeed. If the airspeed is to be reduced in order to maintain a scheduled block time, the fuel consumption will also decrease until the best-range airspeed for that particular tailwind is reached; further airspeed reductions will progressively increase the fuel consumption.

In the presence of a headwind, relatively large increases in the airspeed are required and the improvement in range (fuel savings) is appreciably larger than that for a tailwind, but does not become significant until the wind fraction becomes of the order of -0.3 or larger. The increased airspeed reduces the flight time and thus the block time. For a given headwind, a decrease in the no-wind best-range airspeed is accompanied by an increase in the wind fraction, leading to higher airspeeds and larger relative ranges. In addition, the larger the specified range is, the greater the fuel savings will be. For each wind fraction, there is a relative best-range airspeed which yields the maximum benefits but which may not be operationally attainable; in such a situation, fly as fast as possible.

### References

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